A Comparison of a Hyper-heuristic and an EA on the Portfolio Optimisation Problem

Rahul Soni[1], Matthew J. Craven[1], and David J. Walker[2]

[1] University of Plymouth, Drake Circus, Plymouth, UK
rahul.soni@plymouth.ac.uk
[2] University of Exeter, North Park Road, Exeter, UK

Abstract. We analyse a selection hyper-heuristic (SHH) and NSGA-II for the multi-objective cardinality constrained portfolio optimisation problem, an NP-hard problem addressing the asset allocation trade-off between return and risk under constraints of the number of assets. We evaluated the performance of the SHH and NSGA-II for cardinality constraints $K = \{2, 5\}$. Our results are competitive with those of NSGA-II.

Keywords: Evolutionary algorithms, Hyper-heuristics, Metaheuristics, Combinatorial problems, Portfolio optimisation

The constrained portfolio optimisation problem involves simultaneous optimisation of return and risk with fixed constraints on the number of assets and their individual proportions, typically including a budget constraint ensuring the sum of asset weights equals one. Given a solution (portfolio) $\mathbf{x}$ where x_i indicates the proportion invested in a specific asset, return and risk are optimised with

$$V_p = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \qquad (1) \qquad\qquad R_p = \sum_{i=1}^{N} x_i \mu_i \qquad (2)$$

subject to full investment: $\sum_{i=1}^{N} x_i = 1$. Constraints limit the number of portfolio assets (3) and require a minimum level of investment in any asset (4):

$$\sum_{i=1}^{N} Z_i \leq K \qquad (3) \qquad\qquad x_i \geq x_{\min} \qquad (4)$$

$Z_i = 0$ indicates that asset i has zero weight and $Z_i = 1$ otherwise. The set of all non-dominated solutions to the above problem is called the Pareto front (known as the Cardinality Constrained Efficient Frontier – CCEF – in finance).

We compare the performance of our SHH with that of NSGA-II on problems in which the minimum proportion of investment in selected assets is constrained. The SHH combines choice function-based LLH selection with the solution selection process used in NSGA-II [2]. The strategy dynamically selects from a set of five mutation LLHs (polynomial, Gaussian, ruin and recreate, and two based on a uniform distribution). Cardinalities $K = \{2, 5\}$ were examined, and we varied the minimum proportion values $x_{\min} = 0.1, 0.2, \ldots, 1/K$ to assess the impact of varying minimum proportions on solution quality and algorithm robustness.

SHH and NSGA-II were evaluated on portfolio benchmark datasets [3,4]. Some examples of CCEF approximations produced are given in Fig. 1. Results of

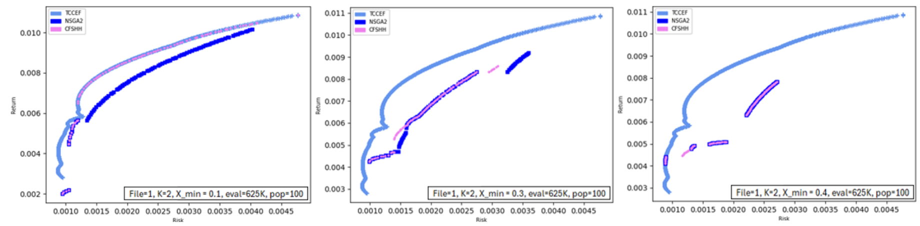

Fig. 1. EFs and approximations for the Hong Kong Hang Seng dataset, for $K = 2$ and $X_{\min} = 0.1, 0.3, 0.4$. Light blue is the true CCEF [4], with solutions produced by NSGA-II in dark blue. Solutions produced by our SHH approach are in pink.

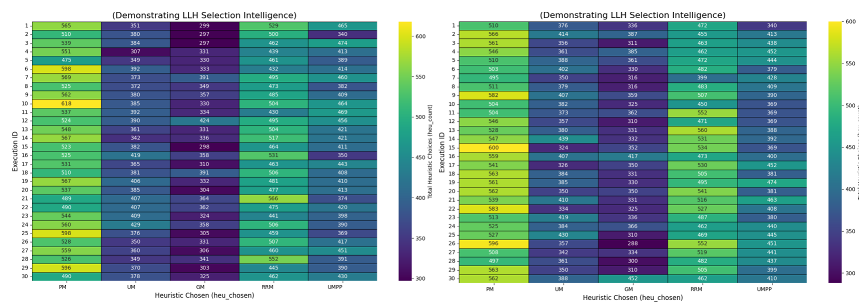

Fig. 2. LLH usage within SHH for the Hong Kong Hang Seng dataset, with $K = 2$, $X_{min} = 0.3$ (left) and 0.4 (right). 'Execution ID' gives the run number.

NSGA-II (blue) generally seem less effective than those of SHH (pink) for these instances and parameters: the SHH approximation dominates that of NSGA-II, being visually closer to the true CCEF particularly for low-to-medium risk. The performance is attributed to SHH's adaptive framework, which efficiently navigates the complex problem search space. Analysis of heuristic usage (Fig. 2) reveals that SHH intelligently switches between different mutation operators, leveraging their strengths at different stages of the optimisation process.

Future work will expand the range of LLHs examined so the effect of different operators can be more fully characterised, as well as considering portfolio problems with a more extensive set of characteristics (e.g., many-objectives).

References

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